

### The greatest integer value of x

Find the greatest integer solution of the equation

<https://www.linkedin.com/groups/8313943/8313943-6379265674996441091>

$$\lim_{n \rightarrow \infty} \frac{n^x - (n-1)^x}{(n+1)^{x-1} + (n+2)^{x-1}} = 2018.$$

### Solution by Arkady Alt, San Jose, California, USA.

I want to note that answer remains the same if in formulation of the problem remove word "integer" and herewith calculation of this limit for any real  $x > 0$  becomes more meaningful problem. Namely, since

$$\frac{n^x - (n-1)^x}{(n+1)^{x-1} + (n+2)^{x-1}} = \frac{(n-1)^x}{(n+1)^x} \frac{(n+1)x \ln\left(\frac{n}{n-1}\right)}{1 + \left(1 + \frac{1}{n+1}\right)^{x-1}} \cdot \frac{e^{x \ln\left(\frac{n}{n-1}\right)} - 1}{x \ln\left(\frac{n}{n-1}\right)}$$

$$\text{and } \lim_{n \rightarrow \infty} \frac{e^{x \ln\left(\frac{n}{n-1}\right)} - 1}{x \ln\left(\frac{n}{n-1}\right)} = 1, \lim_{n \rightarrow \infty} \left(1 + \left(1 + \frac{1}{n+1}\right)^{x-1}\right) = 2, \lim_{n \rightarrow \infty} \frac{(n-1)^x}{(n+1)^x} = 1,$$

$$\lim_{n \rightarrow \infty} (n+1)x \ln\left(\frac{n}{n-1}\right) = x \ln\left(\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n-1}\right)^{n+1}\right) = x \text{ then}$$

$$\lim_{n \rightarrow \infty} \frac{n^x - (n-1)^x}{(n+1)^{x-1} + (n+2)^{x-1}} = \frac{x}{2} \text{ and, therefore, } x = 4036.$$